Optical bistability via coherent and incoherent fields in an Er$^{3+}$-doped yttrium–aluminum–garnet crystal

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**A B S T R A C T**

We investigated the optical bistability in an Er$^{3+}$-doped yttrium–aluminum–garnet (YAG) crystal inside a unidirectional ring cavity. We find that the intensity and the frequency detuning of the coherent field as well as the rate of incoherent field can affect the optical bistability dramatically, which can be used to manipulate efficiently the threshold intensity and the hysteresis loop. The effect of the cooperation parameter on the OB is also studied.

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**1. Introduction**

In the past few decades, lots of quantum optical phenomena based on coherence and quantum interference have attracted attention of many researchers in the quantum coherent media [1–16]. One of the interesting phenomena, the optical bistability (OB) in multilevel atoms confined in an optical cavity [17–34], has been the subject of many recent studies because of its potential wide applications in all-optical switches, memories, transistors, and logic circuits [35–38]. These studies show that one can control the bistable threshold intensity and the hysteresis loop via different approaches such as the field-induced transparency [23], the phase fluctuation [24,25], the squeezed state field [26–28], the spontaneously generated coherence [29–32], the atomic cooperation parameter [33], the intensity of the microwave field [34], and so on.

It should be worth pointing out that many kinds of nonlinear quantum optical phenomena based on the quantum interference and coherence have also been extensively studied in the rare-earth-metal-ion-doped crystals in recent years [39–53], such as Electromagnetically Induced Transparency [39–42], Optical Switch [43,44], Light Storage [45,46], Four-Wave Mixing [47], and Large Refractive Index Without Absorption [48]. The reason for this is mainly the phenomena in these crystals have many potentially important applications in optoelectronics and solid-state quantum information science. Otherwise, these crystals have many inherent characteristics that the atomic systems do not have, such as long coherence times of optical [49–51] and hyperfine transitions [39,49], and optical controllability of the ionic states.

In this work, we investigate the optical bistability in an Er$^{3+}$-doped YAG crystal inside a unidirectional ring cavity. The optical bistability in solid-state semiconductor quantum wells confined in an optical cavity has been studied by Xiao [54] and Li [55] quite recently, however, the optical bistability in the Er$^{3+}$-doped YAG crystal inside a unidirectional ring cavity has never been investigated to our best knowledge, which motivate us to carry out the current work. Our study and the system are mainly based on the Refs. [48,54,55], but our work is very different from those works. First of all, the incoherent field is firstly used to control the optical bistability, and the quantum-interference effect is created by coherent and incoherent fields. Namely, the underlying mechanism is very different from the conventional schemes. Thus, it may provide some new possibilities for technological applications in solid-state quantum information science. Secondly, our scheme have many advantages that the other quantum coherent media do not have: (i) rare-earth-doped crystals have properties similar to atomic vapours but with the advantage of no atomic diffusion; (ii) yttrium aluminum garnet is an excellent optical host material compared with others which are sensitive to moisture; and (iii) the Er$^{3+}$-doped YAG crystal is an efficient active medium for lasers operating in the middle infrared band, coinciding with the telecommunications band [48,52,53]. Thirdly, our study is much more practical than its gaseous counterpart due to its flexible design and the wide adjustable parameters. Therefore, our scheme is very convenient for experimental realization. Our paper is organized as follows: in Section 2, we present the theoretical model and establish the corresponding equations. The experimental realization and numerical results are shown in Sections 3 and 4, respectively. In Section 5, some simple conclusions are given.
2. The model and the dynamic equations

We consider a four-level Er\(^{3+}\) ionic system in an Er\(^{3+}\)-doped YAG crystal as shown in Fig. 1 (see Ref. [48]). The transition \(|2\rangle \leftrightarrow |4\rangle\) is mediated by a coherent coupling field \(E_c\) (frequency \(\omega_c\)), while an incoherent pumping field \(\Lambda\) (rate \(\Gamma\)) and a probe field \(E_p\) (frequency \(\omega_p\)) are applied to the transitions \(|3\rangle \leftrightarrow |1\rangle\) and \(|2\rangle \leftrightarrow |1\rangle\), respectively. If we take the level \(|1\rangle\) as the energy origin and choose the \(\hbar = 1\) [11, 48] in Eq. (3) of the above density-matrix equations [48, 57], which are derived for a system with two resonant frequencies which associates with the corresponding optical transitions \(|2\rangle \leftrightarrow |1\rangle\) and \(|4\rangle \leftrightarrow |2\rangle\), respectively.

By adopting the standard approach [48, 56], the density-matrix equations of motion in dipole and rotating-wave approximations for this system can be written as follows:

\[
\dot{\rho}_{11} = \Omega_{42}^\dagger |4\rangle \langle 2| - \Gamma |1\rangle \langle 1| \rho_{11} + \gamma \rho_{11} \rho_{42} - \gamma \rho_{42} \rho_{11},
\]

\[
\dot{\rho}_{21} = \Omega_{42} |4\rangle \langle 2| - \Omega_{42}^\dagger |2\rangle \langle 4| + \gamma \rho_{21} - \gamma |2\rangle \langle 4| \rho_{21} + \gamma |4\rangle \langle 2| \rho_{21},
\]

\[
\dot{\rho}_{42} = \Omega_{42}^\dagger |4\rangle \langle 2| - \Omega_{42} |2\rangle \langle 4| + \gamma \rho_{42} - \gamma |4\rangle \langle 2| \rho_{42},
\]

\[
\dot{\rho}_{43} = -\Omega_{43} |4\rangle \langle 3| + \gamma |4\rangle \langle 3| \rho_{43} - \gamma \rho_{43} \rho_{43}.
\]

In contrast to many atomic schemes, the transition \(|2\rangle \leftrightarrow |1\rangle\) as the energy origin and choose the \(\hbar = 1\) [11, 48] in Eq. (3) of the above density-matrix equations [48, 57], which are derived for a system with two resonant frequencies which associates with the corresponding optical transitions \(|2\rangle \leftrightarrow |1\rangle\) and \(|4\rangle \leftrightarrow |2\rangle\), respectively.

By adopting the standard approach [48, 56], the density-matrix equations of motion in dipole and rotating-wave approximations for this system can be written as follows:

\[
\dot{\rho}_{11} = \Omega_{42}^\dagger |4\rangle \langle 2| - \Omega_{42} |2\rangle \langle 4| - \Gamma |1\rangle \langle 1| \rho_{11} + \gamma |1\rangle \langle 1| \rho_{42} - \gamma \rho_{42} \rho_{11},
\]

\[
\dot{\rho}_{21} = \Omega_{42}^\dagger |4\rangle \langle 2| - \Omega_{42} |2\rangle \langle 4| + \gamma |2\rangle \langle 4| \rho_{21} - \gamma |2\rangle \langle 2| \rho_{21},
\]

\[
\dot{\rho}_{42} = \Omega_{42}^\dagger |4\rangle \langle 2| - \Omega_{42} |2\rangle \langle 4| + \gamma |4\rangle \langle 2| \rho_{42} - \gamma |4\rangle \langle 4| \rho_{42},
\]

\[
\dot{\rho}_{43} = -\Omega_{43} |4\rangle \langle 3| + \gamma |4\rangle \langle 3| \rho_{43} - \gamma \rho_{43} \rho_{43}.
\]

\[
\rho_{mn} = \rho_{nm}^* (m, n = 1, 2, 3, 4),
\]

and incostrained by \(\rho_{11} + \rho_{42} + \rho_{33} + \rho_{44} = 1\). The total decay rates \(\Gamma_{mn} (m \neq n)\) are added phenomenologically in the above density-matrix equations [48, 57], which are comprised of a population spontaneous emission contribution as well as a dephasing contribution. The first contribution \(\gamma_{mn} (m \neq n)\) designates the population spontaneous damping from \(|m\rangle\) to \(|n\rangle\), while the other contribution \(\gamma_{mn} (m \neq n)\), determined by electron-electron, interface roughness, and photon scattering processes [57], is the dephasing decay rate of the quantum coherence of the \(|m\rangle\) \rightarrow \(|n\rangle\) transition. In contrast to many atomic schemes, the \(\gamma_{mn}^\dagger\) is the dominant contribution to the total decay rate \(\Gamma_{mn}\), which is the major obstacle to the observation of coherent effects such as EIT in solid material media [48].

The behavior of the above-described four-level Er\(^{3+}\) ionic system will be investigated in the unidirectional cavity (see Fig. 1(b)). As we know, the probe field circulates in the ring cavity while the other fields do not circulate in the ring cavity. So, the dynamics of the probe field in the optical cavity is governed by Maxwell's equation, which, under slowly varying envelope approximation, is given by:

\[
\frac{\partial E_p}{\partial t} + \frac{c}{\varepsilon_0} \frac{\partial E_p}{\partial z} = i \frac{\omega_p}{2\varepsilon_0} P(\omega_p),
\]

where \(P(\omega_p)\) is the slowly oscillating term of the induce polarization in the transition \(|2\rangle \leftrightarrow |1\rangle\), which is given by \(P(\omega_p) = N_i \varepsilon_0 \varepsilon_p |\phi_{21}\rangle\), and \(N\) is the number density of the Er\(^{3+}\) ions in the sample. The \(\omega_0\) and \(c\) are the permittivity of free space and the light speed, respectively.

In the steady state, the term \(\frac{\partial E_p}{\partial t}\) in Eq. (3) is equal to zero. Then Eq. (3) can easily be given as follows:

\[
\frac{\partial E_p}{\partial z} = i \frac{N_i \omega_p}{2\varepsilon_0} |\phi_{21}\rangle.
\]
For a perfectly tuned ring cavity, in the steady state limit, the boundary conditions impose the following conditions between the incident field $E_p$ and the transmitted field $E'_p$ [38]

$$E_p(l) = E_p^r / \sqrt{T},$$

$$E_p(0) = \sqrt{T} E_p^i + R E'_p(l),$$

where $L$ is the length of the Er$^{3+}$-doped yttrium–aluminum–garnet crystal sample, and the second term on the right-hand side of Eq. (6) describes a feedback mechanism due to the mirror, which is essential to give rise to OB, that is to say, there is no OB when $R = 0$.

In the mean-field limit [58,59], normalizing the fields by letting $y = \mu E_p^i / h \sqrt{T}$ and $x = \mu E_p^i / 2 h \sqrt{T}$ and using the boundary conditions, we can obtain the input–output relationship:

$$y = 2x - i C \rho_{21},$$

where $C = L N \omega_0 |\mu|^2 / 2 h c c_0 T$ is the usual cooperation parameter. The second term on the right-hand side of Eq. (7) is very important to achieve the OB.

3. Experimental realization

For the experimental realization, we would like to mention some points of the Er$^{3+}$-doped yttrium–aluminum–garnet crystal for the present study, which are given as follows:

(I) The experimental system for this scheme can be realized by Er$^{3+}$-doped YAG crystal with $|1_{1/2}, 1_{1/2}, 1_{1/2}, 1_{1/2}, |$ and $|1_{3/2} 2_1 2_2 |$ behaving the $|1, 2, 3, 4 \rangle$ state labels, respectively. The states $|1, 2, 3 \rangle$ form a usual V-type three-level EIT subsystem, while the states $|1, 2, 3 \rangle$ form a usual ladder-type three-level EIT subsystem [48].

(II) Based on Refs. [48,60], we can get population spontaneous rates $\gamma_m$ of the Er$^{3+}$ ions in Er$^{3+}$-doped YAG crystals with concentrations of Er$^{3+}$ ions at room temperature. So it is reasonable that we can choose the parameters as $\gamma_1 = 239.1$ s$^{-1}$, $\gamma_1 = 0.8 \gamma_21$, $\gamma_1 = 0.8 \gamma_{21}$, $\gamma_22 = 10 \gamma_21$, $\gamma_42 = 0.29 \gamma_21$, and $\gamma_43 = 0.04 \gamma_21$ in the following numerical calculations.

(III) Following Refs. [48,50], we have found the dephasing time of Er$^{3+}$-doped YAG crystal with an Er$^{3+}$ concentration of 0.1%, $\Omega = 75 \mu$s on the $|1_{1/2} 2_1 2_2 |$ transition of Er$^{3+}$ at 1526.97 nm, the homogeneous linewidth $\Gamma_h = 4.286$ kHz. So it is reasonable for us to estimate the dephasing decay rate as $\gamma_{\text{dph}} = \gamma_1 = \gamma_21 = \gamma_32 = \gamma_43 = 1526.97$, and $\gamma_43 = 1526.97$.

According to the above conditions, we believe that the experimental scientists have adequate wisdom to realize our scheme.

4. Numerical results

Now we give some numerical studies for the steady-state of the output intensity $|y|$ versus the input intensity $|x|$ with different corresponding parameters, as shown in Figs. 2–5. In the following numerical calculations, all the parameters will be scaled by $y = \gamma_21$ and choose the decay rates as $\gamma_1 = 0.8 \gamma, \gamma_3 = 0.8 \gamma, \gamma_22 = 10 \gamma, \gamma_42 = 0.29 \gamma, \gamma_43 = 0.04 \gamma$, and $\gamma_{\text{dph}} = \gamma_1 = \gamma_21 = \gamma_32 = \gamma_43 = 15 \gamma$, all of which will not be given again in Figs. 2–5.

First of all, we will analyze how the rate of incoherent pumping field $\Lambda$ affects the optical bistability, while keeping all other parameters fixed. Figs. 2(a) and 3(a) demonstrate the dependence of the optical bistability on the rate of incoherent pumping field. For a small rate of incoherent pumping field, say $\Lambda = 0.7 \gamma$ as shown in Fig. 2(a), we find that threshold reduces dramatically due to the increasing rate of incoherent pumping field, while increasing $\Lambda$ further to a large value, say $\Lambda = 3 \gamma$ as shown in Fig. 3(a), it can be easily seen from Fig. 3(a) that the threshold increases obviously with the increasing rate of incoherent pumping field. The reasons for the above results can be qualitatively explained as follows. It is known that states $|1, 2, 3 \rangle$ and $|1, 2, 3 \rangle$ are comprised of a standard V-type three-level EIT system, by applying an increasingly incoherent pumping field between the states $|1, 2, 3 \rangle$ and $|1, 2, 3 \rangle$, we can modify the absorption for the weak probe field on the transition 2 to 1 and affect the Kerr nonlinearity of the Er$^{3+}$-doped YAG crystal medium, which makes the hysteresis cycle changes dramatically.

In order to gain deeper insight into the phenomena, we plot the absorption $\text{Im}(\rho_{21})$ and dispersion $\text{Re}(\rho_{21})$ versus the frequency detuning of probe field in Fig. 2(b), and show the absorption $\text{Im}(\rho_{21})$ and dispersion $\text{Re}(\rho_{21})$ versus the rate of incoherent pumping field in Fig. 3(b), respectively. We can easily see from Figs. 2(b) and 3(b) that the dispersion is equal to zero all the time, but the absorption (at the absorptive line center $\Delta_p = 0$) reduces in Fig. 2(b) and increases (after point A) in Fig. 3(b). So, we can see the threshold of OB goes down in Fig. 2(a) but goes up in Fig. 3(a).

In the following, we display the influence of the coherent coupling field $\Omega_e$ on the optical bistability in Fig. 4. Clearly, the increasing intensity of the coherent coupling field leads to a significant decrease of the bistable threshold in Fig. 4(a). A reasonable explanation for this is that, under the resonance condition, the varying coherent coupling...
field will alter the absorption and nonlinearity of the YAG crystal medium greatly. In fact, the states $|1\rangle$, $|2\rangle$, and $|4\rangle$ form a usual ladder-type three-level EIT system at this time, the strong coherent coupling field will sharply suppress the absorption of the Er$^{3+}$-doped YAG crystal medium [48]. We also plot the absorption $\text{Im}(\rho_{21})$ and dispersion $\text{Re}(\rho_{21})$ versus the frequency detuning of the probe field in Fig. 4(b). We find that the absorption at the absorptive line center is dramatically suppressed by the increasing intensity of the coherent coupling field, and the dispersion (at the dispersive line center $\Delta_p = 0$) is equal to zero all the same. As a result, we can see that the bistable threshold goes down.

Finally, the effects of the frequency detuning of the coherent coupling field $\Delta_p$ and the cooperation parameter $C$ on the optical bistability are depicted in Fig. 5. It is seen from Fig. 5(a) that the threshold of OB is sensitive to the frequency detuning of the coherent coupling field, which is the result of the small influence of the frequency detuning of the coherent coupling field on the absorption and nonlinearity properties of the Er$^{3+}$-doped YAG crystal medium when the intensity of coherent coupling field is not very strong. In Fig. 5(b), we show the dependence of the optical bistability on the cooperation parameter. When the cooperation parameter becomes larger, we can see that the bistable threshold goes up. From the term $C = \frac{L\text{Im}(\mu_0)}{2\hbar c_0 T}$, we can see that the cooperation parameter is directly proportional to the number density of Er$^{3+}$ ions, and the increase of the number density of Er$^{3+}$ ions will enhance the absorption of the sample, which accounts for the raise of the bistable threshold.

5. Conclusions

To sum up, we have theoretically investigated the optical bistability in an Er$^{3+}$-doped yttrium–aluminum–garnet (YAG) crystal inside a ring cavity. We find that the intensity and the frequency detuning of the coherent field as well as the rate of the incoherent field can affect the optical bistability dramatically, which can be used to manipulate efficiently the threshold intensity and the hysteresis loop. The effect of the cooperation parameter on the OB is also studied. By controlling the threshold intensities and the hysteresis loop of OB with these different physical parameters, we can build more efficient all-optical switches and logic-gate devices for optical computing and quantum information processing [21] in such crystal medium which is much more practical than that in gaseous medium because of its...
flexible design and the wide adjustable parameters. The underlying mechanism is very different from the conventional schemes. Thus, it may provide some new possibilities for technological applications.

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